

Roll No. ____ (To be filled in by the candidate) (Academic Sessions 2021 - 2023 & 2023 - 2025)

225-1st Annual-(INTER PART - II)

MATHEMATICS

Group - II

Time Allowed : 30 Minutes

Q. Paper - II (Objective Type)

Paper Code = 8196

Maximum Marks: 20

NOTE: Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answers book. Cutting or filling two or more circles will result in zero mark in that question.

Q1.

1	$\frac{d}{dx}(e^{\tan x}) =:$ (A) $e^{\tan x} \sec^2 x$ (B) $e^{\tan x}$ (C) $e^{\tan x} \cdot \ln \sec^2 x$ (D) $e^{\tan x} \ln \tan x$
2	$\int \frac{x^4 - 1}{x^2 - 1} dx =:$ (A) $\frac{x^3}{3} - x + c$ (B) $\frac{x^4}{4} + x + c$ (C) $\frac{x^5}{5} - x + c$ (D) $\frac{x^3}{3} + x + c$
3	The range of $f(x) = \sqrt{x^2 - 9}$ (A) $(-\infty, 0]$ (B) $(0, +\infty]$ (C) $[0, +\infty)$ (D) $(-\infty, +\infty]$
4	$\int e^x [f(x) + f'(x)] dx =:$ (A) $e^x f(x) + c$ (B) $e^x f(x) + x + c$ (C) $e^x + e^x f'(x) + c$ (D) $e^x f'(x) + c$
5	What is the value of $\lim_{x \rightarrow 0} (x \sin x)$: (A) α (B) -1 (C) 1 (D) 0
6	$\int \frac{1}{x \ln x} dx =:$ (A) $\ln x + c$ (B) $\ln \ln(\ln x) + c$ (C) $\ln(\ln x) + c$ (D) $\ln x^2 + c$
7	$\frac{d}{dx}(\tanh^{-1} x) =:$ (A) $\frac{1}{1+x^2}$ (B) $\frac{1}{1-x^2}$ (C) $\frac{1}{\sqrt{1+x^2}}$ (D) $\frac{1}{\sqrt{1-x^2}}$
8	$\frac{d^2}{dx^2}(\cosh 3x) =:$ (A) $3 \cos h 3x$ (B) $3 \sin 3x$ (C) $-\cos h 3x$ (D) $9 \cosh 3x$
9	$\int \frac{\cos(\ln x)}{x} dx =:$ (A) $\cos(\ln x) + c$ (B) $-\cos(\ln x) + c$ (C) $-\sin(\ln x) + c$ (D) $\sin(\ln x) + c$
10	Any point where function is neither increasing nor decreasing provided $f'(x) = 0$ is called: (A) Critical point (B) Point of inflection (C) Stationary point (D) Feasible point
11	Axis of parabola $x^2 = -4ay$ is: (A) $x = 0$ (B) $y = 0$ (C) $x = -1$ (D) $x = 1$
12	The general equation of the straight line in two variables x and y is: (A) $ax + by + c = 0$ (B) $ax^2 + by + c = 0$ (C) $ax + by^2 + c = 0$ (D) $ax^2 + by^2 + c = 0$
13	If a and b have the opposite directions then $a \cdot b =:$ (A) $-ab$ (B) ab (C) $ab \sin \theta$ (D) $ab \tan \theta$
14	Equation of latus rectum of the parabola $x^2 = 4ay$ is: (A) $x = a$ (B) $y = a$ (C) $x = -a$ (D) $y = -a$
15	If u is a vector such that $u \cdot i = 0$, $u \cdot j = 0$, $u \cdot k = 0$, then u is: (A) Unit vector (B) Null vector (C) Special vector (D) Triple vector
16	The point $(-2, 4)$ lies the line $2x + 5y - 3 = 0$: (A) On (B) Below (C) Above (D) Near
17	$x = 0$ is the solution of the inequality: (A) $2x + 1 > 0$ (B) $2x + 1 < 0$ (C) $2x + 1 \leq 0$ (D) $2x + 4 < 0$
18	If $u = 3i - j + 2k$ then $[3, -1, 2]$ are called: (A) Direction cosines (B) Direction ratios (C) Direction angles (D) Elements
19	The slope of the line $ax + by + c = 0$ is: (A) $\frac{b}{a}$ (B) $-\frac{a}{b}$ (C) $\frac{a}{b}$ (D) $-\frac{b}{a}$
20	A line through the focus perpendicular to the directrix is known as: (A) Dirextrix (B) Axis of conic (C) Latus lectum (D) Fixed straight line

SECTION - I

Q2. Write short answers to any EIGHT (8) questions:

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- (i) Show that the parametric equations $x = a \sec \theta$, $y = b \tan \theta$ represent the equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
 (ii) Without finding the inverse, state the domain and range of f^{-1} , where $f(x) = 2 + \sqrt{x-1}$ (iii) Define continuity of a function at a number C. (iv) Evaluate $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin \theta}$ (v) Find $f \circ g(x)$ and $g \circ f(x)$, when $f(x) = 2x + 1$ and $g(x) = \frac{3}{x-1}$, $x \neq 1$ (vi) Find $\frac{dy}{dx}$, if $x = at^2$ and $y = 2at$. (vii) Prove that $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$, $\forall x \in \mathbb{R}$
 (viii) If $x = y \sin y$, find $\frac{dy}{dx}$ (ix) Find y_2 , if $y = \ell n \left(\frac{2x+3}{3x+2} \right)$ (x) Apply the Maclaurin Series expansion to prove that $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$ (xi) Determine the intervals in which f is increasing or decreasing, $f(x) = \cos x$; $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$ (xii) Differentiate $x^2 - \frac{1}{x^2}$ w.r.t x^4

Q3. Write short answers to any EIGHT (8) questions:

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- (i) Find δy of the function $y = x^2 - 1$ when x changes from 3 to 3.02.
 (ii) Evaluate $\int \frac{\sin \theta}{1 + \cos^2 \theta} d\theta$ (iii) Find $\int x \sin x dx$ (iv) Evaluate the integral $\int \frac{1}{x \ell n x} dx$ (v) Find the definite integral $\int_1^2 \frac{x}{x^2 + 2} dx$ (vi) Evaluate $\int \frac{e^{\tan^{-1} x}}{1 + x^2} dx$
 (vii) Solve the differential equation $(x-1)dx + y dy = 0$ (viii) Define inclination of a line. (ix) Check whether the lines $4x - 3y - 8 = 0$, $3x - 4y - 6 = 0$ and $x - y - 2 = 0$ are concurrent. (x) Find an equation of line through A(-6, 5) having slope 7. (xi) Convert $15y^2 - 8x + 7 = 0$ into normal form of line. (xii) Define homogeneous function.

Q4. Write short answers to any NINE (9) questions:

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- (i) Write the procedure for graphing a linear inequality in two variables. (ii) Draw a graph of linear inequality $x + 2y < 6$ (iii) Show that the equation $5x^2 + 5y^2 + 24x + 36y + 10 = 0$ represents a circle. Also find its centre and radius. (iv) Check the position of point (5, 6) with respect to circle $2x^2 + 2y^2 + 12x - 8y + 1 = 0$ (v) Write an equation of parabola with focus (1, 2) and vertex (3, 2) (vi) Find foci and eccentricity of ellipse $9x^2 + y^2 = 18$ (vii) Find the centre and direction of hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ (viii) Find a unit vector in the direction of the vector $\underline{y} = 2\hat{i} + 6\hat{j}$ (ix) Find the vector from the point A to the origin where $\overline{AB} = 4\hat{i} - 2\hat{j}$ and B is the point (-2, 5) (x) Find a and b so that the vectors $3\hat{i} - \hat{j} + 4\hat{k}$ and $a\hat{i} + b\hat{j} - 2\hat{k}$ are parallel. (xi) Find the projection of $\underline{a}$ along $\underline{b}$ when $\underline{a} = 3\hat{i} + \hat{j} - \hat{k}$, $\underline{b} = -2\hat{i} - \hat{j} + \hat{k}$ (xii) Find a vector perpendicular to vectors $\underline{a} = 2\hat{i} + \hat{j} + \hat{k}$, $\underline{b} = 4\hat{i} + 2\hat{j} - \hat{k}$ (xiii) Differentiate between scalar quantity and vector quantity.

Note: Attempt any THREE questions.

SECTION - II

- Q5. (a) If $f(x) = \begin{cases} 3x & \text{if } x \leq -2 \\ x^2 - 1 & \text{if } -2 < x < 2 \\ 3 & \text{if } x \geq 2 \end{cases}$ (b) Discuss continuity at $x = 2$ and $x = -2$ 5,5
 Q6. (a) Differentiate $\frac{x^2 + 1}{x^2 - 1}$ w.r.t. $\frac{x-1}{x+1}$ 5
 (b) Find the extreme values of the function defined by $f(x) = x^3 - 6x^2 + 9x$ 5
 Q7. (a) Evaluate the integral $\int \frac{5x+8}{(x+3)(2x-1)} dx$ (b) Evaluate $\int_0^{\frac{\pi}{2}} \cos^4 t dt$ 5,5
 Q8. (a) Find a joint equations of the lines through the origin and perpendicular to the lines $x^2 - 2xy \tan \alpha - y^2 = 0$ 5
 (b) Find an equation of circle which passes through the points A(5, 10), B(6, 9) and C(-2, 3) 5
 Q9. (a) If $\underline{a} = 4\hat{i} + 3\hat{j} + \hat{k}$ and $\underline{b} = 2\hat{i} - \hat{j} + 2\hat{k}$, find a unit vector perpendicular to both $\underline{a}$ and $\underline{b}$. Also find the sine of angle between them. 5
 (b) Show that an equation of the parabola with focus at $(a \cos \alpha, a \sin \alpha)$ and directrix $x \cos \alpha + y \sin \alpha + a = 0$ is $(x \sin \alpha - y \cos \alpha)^2 = 4a(x \cos \alpha + y \sin \alpha)$ 5

Note: For Answers of long questions please study "Hamdard Notes Mathematics" Class 12.